CONJUGATE PROBLEM OF NONSTATIONARY HEAT TRANSFER WITH BLOWING

L. B. Gdalevich

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A solution is presented for the problem of conjugate nonstationary heat transfer in a laminar boundary layer on the boundary of a semiinfinite porous medium when the blowing velocity varies with time as $t^{-1/2}$.

Blowing in a boundary layer results in 1) direct cooling of the body surface as a result of heat transfer to the coolant and 2) degradation of the heat transfer between the body and the main flow.

Traditionally, heat transfer in the boundary layer on a permeable surface was examined without taking account of the influence of the porous body; hence, the boundary conditions on the interphasal body–fluid surface were considered known beforehand [1]. To take account of the connection between the heat transfer in the boundary layer and the porous body, it is necessary to solve jointly the boundary layer equation on a permeable interphasal body–fluid surface and the heat conduction equation within the porous body (i.e., formulate a conjugate problem [2]).

Few authors examined the problem of boundary layer control in a conjugate formulation. The effect of "underheating" of a fluid at the surface of a longitudinally streamlined plate caused by homogeneous suction was computed in [3]. Two heat-transfer problems in a laminar boundary layer on the boundary of a porous medium ($0 < x < \infty$, $-\infty < y < 0$) when the blowing velocity is proportional to $x^{-1/2}$ are solved in [4]. Either convective heat transport or heat conduction in the main-stream direction was taken into account in the body. By using the equation of heat conduction in a porous body derived in [6], the author graphically showed in [5] the influence of the heat conductivity of a porous plate ($-\infty \le x \le \infty$, $0 \le y \le h$) on the heat transfer in Couette hot-gas flow on its surface when $\rho v|_{\omega}$ = const. A method was proposed in [7] for the computation of the cooling of a porous wall ($0 \le x \le \infty$, $0 \le y \le h$) in a turbulent compressible gas flow. The heat conduction of the coolant was considered negligible as compared to the heat conduction of the wall material, whose value in the main-stream direction is negligible in comparison to the value in the transverse direction. The authors discarded the usual assumption about equality of the body and cooling surface temperatures on the interface. Stationary heat transfer was considered in all the problems listed.

Let us consider heat transfer in a body-fluid system consisting of a half-space ($y > 0$) filled with a viscous incompressible fluid and a porous mass ($y < 0$) which is set impulsively in motion with a constant velocity U_0 parallel to the plane $y = 0$.

The equations of a laminar boundary layer with blow have the usual form

$$\partial v/\partial y = 0,\ \partial u/\partial t + v\partial u/\partial y = \nu\partial^2 u/\partial y^2,\ u(y,\ 0) = 0,$$
$$u(0,\ t) = U_0,\ u(\infty,\ t) = 0,\ v(0,\ t) = v_0(t). \tag{1}$$

We assume the coolant to be sucked continuously through the surface $y = 0$, and the speed of the blowing to be proportional to $t^{-1/2}$:

$$v_0(t) = V_0(\nu/t)^{1/2},\ V_0 > 0. \tag{2}$$

Then problem (1) is self-similar, and its solution is [8]

$$u^*(\eta) = \frac{\operatorname{erfc}(\eta - V_0)}{\operatorname{erfc}(-V_0)},$$
$$\eta = y/2(\nu t)^{1/2}. \tag{3}$$

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk.
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To describe the heat transfer in a porous body we use the equation derived in [9]:

$$[c_s\rho_s(1-\psi)+c_f\rho_f\psi]\frac{\partial T}{\partial t}=-\operatorname{div}(-\lambda\operatorname{grad}T)-c_f\rho_f\psi\mathbf{V}^*\operatorname{grad}T, \tag{4}$$

where T is the temperature of an arbitrary point of the porous material; $\mathbf{V}^*$, velocity of the liquid phase in the pores; ψ, porosity factor; k_f (k_s), heat conduction coefficient of the liquid phase (the porous skeleton); $\lambda = (1-A)k_s + Ak_f$, effective heat conduction of the porous material;

$$A=\frac{2^n}{2^n-1}\{1-1/(1+\psi)^n\},\ n>0. \tag{4'}$$

Equation (4) takes account of the effect of the porosity and the flow of the liquid phase through the porous skeleton, and also agrees well with experimental results. This latter was not taken into account by a number of authors [6, 10, 11].

Both in the pores of the skeleton and in the boundary layer the transverse fluid velocity component is a function of just the time and equals $v_0(\nu/t)^{1/2}$, the blowing velocity on the contact surface y = 0. We assume equality of the skeleton and coolant temperatures upon emergence from the pores (for y = 0 [12]). The fluid being blown is homogeneous with the fluid of the main stream. Let us formulate the thermal conjugate problem:

$$c_f\rho_t\left[\frac{\partial T_f}{\partial t}+V_0(\nu/t)^{1/2}\frac{\partial T_f}{\partial y}\right]=k_f\frac{\partial^2T_f}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2,\ y>0,\ t>0, \tag{5}$$

$$[c_s\rho_s(1-\psi)+c_f\rho_f\psi]\frac{\partial T}{\partial t}+c_f\rho_f\psi V_0(\nu/t)^{1/2}\frac{\partial T}{\partial y}=\lambda\frac{\partial^2T}{\partial y^2},\ y<0,\ t>0. \tag{6}$$

The initial conditions (t = 0) are

$$T_f=T_0,\ y>0, \tag{7}$$

$$T=0,\ y<0. \tag{8}$$

The boundary conditions (t > 0) are

$$T_f=T,\ y=0, \tag{9}$$

$$k_f(\partial T_f/\partial y)=\lambda(\partial T/\partial y),\ y=0, \tag{10}$$

$$T_f=T_0,\ y=\infty, \tag{11}$$

$$T=0,\ y=-\infty. \tag{12}$$

Taking account of (3), the dimensionless form of the problem (5)-(12) is

$$\frac{d^2\theta_f}{d\eta^2}+2\Pr(\eta-V_0)\frac{d\theta_f}{d\eta}=-\frac{4\Pr\mathrm{Ec}\exp[-2(\eta-V_0)^2]}{\pi[\operatorname{erfc}(-V_0)]^2}, \tag{13}$$

$$\frac{d^2\theta}{d\eta^2}+\frac{2\Pr}{(1-A)K+A}[(K_{cp}(1-\psi)+\psi)\eta-\psi V_0]\frac{d\theta}{d\eta}=0, \tag{14}$$

$$\theta_f=\theta,\ \eta=0, \tag{15}$$

$$\frac{d\theta_f}{d\eta}=[(1-A)K+A]\frac{d\theta}{d\eta},\ \eta=0, \tag{16}$$

$$\theta_f=1,\ \eta=\infty, \tag{17}$$

$$\theta=0,\ \eta=-\infty, \tag{18}$$

where $u^* = u/U_0$; $\theta_f = T_f/T_0$; $\theta = T/T_0$; $K = k_s/k_f$; $K_{cp} = (c_s\rho_s/c_f\rho_f)$; $\mathrm{Ec} = U_0^2/c_fT_0$, Eckert number; $\Pr = c_f\mu/k_f$, Prandtl number. The solution of (13) satisfying boundary condition (17) has the form [13]

$$\theta_f = -hI(\eta,\ \mathrm{Pr},\ V_0) - \frac{c_1}{2}\left(\frac{\pi}{\mathrm{Pr}}\right)^{1/2} \mathrm{erfc}[\mathrm{Pr}^{1/2}(\eta - V_0)] + 1,$$
$$I(\eta,\ \mathrm{Pr},\ V_0) = \int\limits_{t=\eta}^{\infty} \exp[-\mathrm{Pr}(t - V_0)^2]\, \mathrm{erfc}\, [(2-\mathrm{Pr})^{1/2}(t - V_0)]\, dt, \tag{19}$$

where

$$h = \frac{2\,\mathrm{Pr}\,\mathrm{Ec}}{[\pi(2-\mathrm{Pr})]^{1/2}\,[\mathrm{erfc}(-V_0)]^2}. \tag{20}$$

The solution of (14) satisfying the boundary condition (18) has the form

$$\theta = c_2\, \mathrm{erfc}\, [-(a/2b)^{1/2}(b\eta - \psi V_0)],$$

where

$$a = 2\,\mathrm{Pr}/[(1-A)K + A],\ \ b = K_{\mathrm{c\rho}}(1-\psi) + \psi.$$

We find the constants c_1, c_2 from the conjugate conditions (15)-(16):

$$c_1 = \Delta_1/\Delta,\ \ c_2 = \Delta_2/\Delta,$$

where

$$\Delta_1 = [1 - hI(0,\ \mathrm{Pr},\ V_0)][(1-A)K + A](2ab/\pi)^{1/2} \exp[-a(\psi V_0)^2/2b] -$$
$$- h \exp(-\mathrm{Pr}\, V_0^2)\, \mathrm{erfc}\, [-V_0(2-\mathrm{Pr})^{1/2}]\, \mathrm{erfc}\, [(a/2b)^{1/2}\psi V_0];$$
$$\Delta_2 = \exp(-\mathrm{Pr}\, V_0^2)\{1 - hI(0,\ \mathrm{Pr},\ V_0) + (h/2)(\pi/\mathrm{Pr})^{1/2}\ \mathrm{erfc}(-\mathrm{Pr}^{1/2}V_0)\, \mathrm{erfc}\, [-V_0(2-\mathrm{Pr})^{1/2}]\}; \tag{21}$$
$$\Delta = \exp(-\mathrm{Pr}\, V_0^2)\, \mathrm{erfc}\, [(a/2b)^{1/2}\psi V_0] +$$
$$+ (ab/2\mathrm{Pr})^{1/2}[(1-A)K + A] \exp[-a(\psi V_0)^2/2b]\, \mathrm{erfc}(-\mathrm{Pr}^{1/2}V_0).$$

In the case of an impermeable wall ($\psi = V_0 = 0$), the constants c_1, c_2 are evaluated from the formulas

$$\tilde{c}_1 = \tilde{\Delta}_1/\tilde{\Delta},\ \ \tilde{c}_2 = \tilde{\Delta}_2/\tilde{\Delta},$$
$$\tilde{\Delta}_1 = 2(\mathrm{Pr}KK_{\mathrm{c\rho}}/\pi)^{1/2}[1 - \tilde{h}I(0,\ \mathrm{Pr})] - \tilde{h},$$
$$\tilde{\Delta}_2 = 1 - \tilde{h}I(0,\ \mathrm{Pr}) + (\tilde{h}/2)(\pi/\mathrm{Pr})^{1/2}, \tag{22}$$
$$\tilde{\Delta} = 1 + (KK_{\mathrm{c\rho}})^{1/2},$$
$$\tilde{h} = 2\,\mathrm{Pr}\,\mathrm{Ec}/\pi(2-\mathrm{Pr})^{1/2}.$$

The appropriate solution agrees with the solution in [14].

The integral $I(0,\ \mathrm{Pr},\ V_0)$ in (21) can be expressed in terms of elementary functions if the method of evaluating improper multiple integrals proposed by Poisson is used:

$$I(0,\ \mathrm{Pr},\ V_0) = \int\limits_0^{\infty} \exp[-\mathrm{Pr}(x - V_0)^2]\, \mathrm{erfc}\, [(2-\mathrm{Pr})^{1/2}(x - V_0)]\, dx =$$
$$= \frac{2}{(\pi)^{1/2}} \int\limits_0^{\infty} \exp[-\mathrm{Pr}(x - V_0)^2] \left(\int\limits_{(2-\mathrm{Pr})^{1/2}(x-V_0)}^{\infty} \exp(-y^2)\, dy \right) dx =$$
$$= \frac{2}{(\pi)^{1/2}} \int\limits_0^{\infty} dx \int\limits_{(2-\mathrm{Pr})^{1/2}(x-V_0)}^{\infty} \exp[-\mathrm{Pr}(x - V_0)^2 - y^2]\, dy = \frac{2}{(\pi)^{1/2}} \int\limits_0^{\infty} \int\limits_{(2-\mathrm{Pr})^{1/2}(x-V_0)}^{\infty} \exp[-\mathrm{Pr}(x - V_0)^2 - y^2]\, dxdy.$$

After going over to polar coordinates

$$\begin{cases} x = V_0 + (\mathrm{Pr})^{-1/2} r \cos\varphi, \\ y = r \sin\varphi, \end{cases}$$
$$ds = \left| \frac{D(x,\ y)}{D(r,\ \varphi)} \right| drd\varphi = (\mathrm{Pr})^{-1/2} rdrd\varphi$$

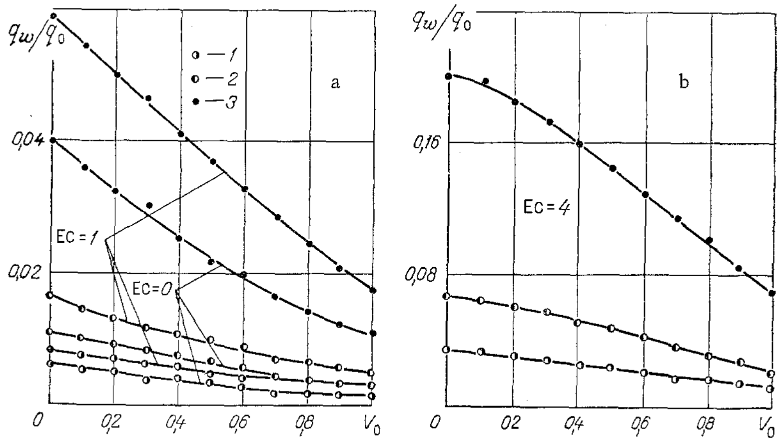

Fig. 1. Dependence of the heat-flux ratio q_ω/q_0 on the blowing for different body–fluid pairs when the Eckert number is Ec = 0, 1, 4: 1) glass (ordinary)–air; 2) rubber–air; 3) aluminum (99%)–air; n = 2 (4'), ψ = 50%.

we obtain

$$I(0,\ \mathrm{Pr},\ V_0) = \frac{2}{(\pi\,\mathrm{Pr})^{1/2}} \int\limits_{\mathrm{arctg}\left(\frac{2-\mathrm{Pr}}{\mathrm{Pr}}\right)^{1/2}}^{\pi/2} \int\limits_{(2-\mathrm{Pr})^{1/2}V_0}^{\infty} \exp(-r^2)\,r\,dr = \left[\frac{\pi}{2} - \mathrm{arctg}\left(\frac{2-\mathrm{Pr}}{\mathrm{Pr}}\right)^{1/2}\right] / (\pi\,\mathrm{Pr})^{1/2} \exp[(2-\mathrm{Pr})\,V_0^2].$$

The heat flux density through the body–fluid interface is proportional to $t^{-1/2}$:

$$q_\omega(t) = k_f(\partial T_f/\partial y)|_{y=0} = -\frac{k_f T_0}{2(\nu t)^{1/2}}\{h\,\mathrm{erfc}[-V_0(2-\mathrm{Pr})^{1/2}] + c_1\}\exp(-\mathrm{Pr}\,V_0^2),$$

and the ratio $q_\omega(t)/q_0(t)$, where $q_0(t)$ is the heat flux density through the body–fluid interface in the case of an impermeable wall, equals

$$q_\omega/q_0 = \exp(-\mathrm{Pr}\,V_0^2)\{h\,\mathrm{erfc}[-V_0(2-\mathrm{Pr})^{1/2}] + c_1\}/(\bar{h} + \bar{c}_1). \quad (23)$$

Numerical computations by using (21)-(23) showed:

1) the dependence of the ratio q_ω/q_0 on the conjugate parameter $\alpha = 2(\mathrm{Pr}KK_{c\rho}/\pi)^{1/2}$ is not monotonic in nature;

2) an increase in the Eckert number is accompanied by growth of the ratio q_ω/q_0;

3) as the blowing velocity $V_0(\psi = \mathrm{const})$ rises, the ratio q_ω/q_0 decreases almost linearly; the decrease will be more abrupt the greater the Eckert number Ec;

4) the ratio q_ω/q_0 hardly changes with the rise in body porosity $\psi(V_0 = \mathrm{const})$.

An example of the computations is presented in Fig. 1.

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HEATING OF A BULKY BODY BY A CIRCULAR HEAT SOURCE WITH HEAT ELIMINATION FROM THE SURFACE TAKEN INTO ACCOUNT

L. É. Melamed

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Results of an analytical and numerical solution of the problem, in a form suitable for the determination of material properties, are given.

The problem of heating a bulky body by a circular heat source is a computational scheme of an enormous number of local-heating cases encountered in engineering. Included here are the electroerosive treatment of metals, electron-beam and laser treatment, welding, the action of local heat sources in a fire, and problems of many other branches of engineering. This research is performed directly in connection with the problem of determining the heat conductivity of structural constructions (panels, etc.) under nondestructive testing – the action of a circular heat source of given intensity on the surface of an item. A stationary modification of such a method is proposed in [1]. The thermal engineering basis of the nonstationary modification of the method, proposed by the same author, is examined below. Particular cases of this computational scheme were examined in [2-6].

The problem is formulated thus. The equation

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2}, \quad r,\ z \geqslant 0, \quad \tau > 0$$

with the boundary conditions

$$\theta = 0 \ \text{for}\ \tau = 0,\ \theta \to 0 \ \text{for}\ r,\ z \to \infty,$$

$$\mathrm{Bi}\,\theta - \frac{\partial \theta}{\partial z} = A(r,\ \tau) \ \text{for}\ z = 0$$

is solved.

In quadratures, the solution of the problem has the form

$$\theta(r,\ z,\ \tau) = \int_0^\tau \int_0^\infty \frac{\zeta A(\zeta,\ \tau - t)}{2t} \exp\left(-\frac{r^2 + z^2 + \zeta^2}{4t}\right) I_0\left(\frac{r\zeta}{2t}\right) \times$$

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